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Exploiting hydrodynamic instabilities. Resonant heat transfer enhancement

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Abstract—We introduce here the concept of resonant heat transfer enhancement based on excitation of shear-layer instabilities present in internal separated flows. Exploitation of natural instabilities requires: creation of a system with separated flow; determination of the system's resonant frequency; and excitation of that frequency with appropriate modulation. The resulting large scale motions lead to significant lateral mixing and correspondingly dramatic heat transfer enhancement. The method is applicable both in laminar and turbulent flows. Results, experimental and numerical, for a subcritical grooved channel flow and a cross flow around a cylinder are presented. For the case of grooved channel, up to three-fold enhancement of heat transfer is observed when the flow is modulated at the system's natural frequency.

1. INTRODUCTION

IN MANY applications it is heat transfer processes that limit the size, performance and efficiency of engineering systems. Primarily for this reason, we have seen in the last several decades an increasing interest in heat transfer enhancement. The first extensive survey on the subject appeared 20 years ago [1], and the most recent guide to the literature was assembled in 1985 [2].

We define augmentation as an increase (enhancement) of the convective heat transfer coefficient produced by deliberate modification of a system. It is understood that the 'enhancement' refers to an improvement relative to the performance of a 'standard', functionally similar system. This definition appears consistent with the past usage and convention and allows for a meaningful comparison between transport enhancement and the requirement for increased pumping power for the two respective systems.

There is presently a broad range of augmentation schemes that have found their way to everyday engineering practice. The proposed schemes are usually grouped according to the type of hardware modification imposed on the standard system [2]. Another possible categorization would be to divide various methods according to the physical effects that they produce. This latter classification leads to only a few (generic) categories. Thus for single-phase systems we can identify the following two:

1. Methods involving *changes in effective thermophysical properties* (molecular and microscopic effects) (such as additive schemes and electrostatic field applications).
2. *Mixing*—defined as exchange of fluid mass normal to the heat transfer surface.

An important class in the second broad category is represented by *mixing produced* by *hydrodynamic instabilities*. It plays an important role in the augmentation schemes both in terms of frequency of its application and its potential for further development.

All turbulent flows represent common examples of mixing produced by hydrodynamic instability. For internal flows without separation the onset of self-sustained mixing (turbulence) requires a finite (nonlinear) perturbation and flow Reynolds number (Re) above some critical value (Re_c). For flow Reynolds numbers below Re_c, any flow perturbation would decay exponentially. For $Re > Re_c$ the process of fluid exchange at the wall is maintained by periodic disruptions of the laminar sublayer (bursts) which are primarily caused by instability of the layer itself. With an increase in Reynolds number, the frequency of the disruptions increases, resulting in a better exchange of fluid near the wall. Although this process would yield higher heat transfer coefficients, it is not considered an augmentation method since it occurs 'naturally' without any modification of the system. On the other hand, introduction of roughness on the heat transfer surfaces, which would further destabilize the sublayer and therefore locally intensify the mixing process, might be considered an augmentation scheme.

In systems with separated flows—a class of flows of considerable interest in augmentation schemes—the behavior is markedly different to that found in smooth-wall channel flows. With the presence of separation, the velocity profile will have an inflection point and consequently there will be (interior to the flow field) a surface with concentrated vorticity, making this region susceptible to the Kelvin–Helmholtz shear layer instability. At a Reynolds number greater than a critical value (which is typically lower than the one marking transition to full turbulence in the channel), self-sustained oscillations at a well defined frequency would set in, forced by the Kelvin–Helmholtz instabilities. The onset of these oscillations does not require a finite perturbation of the flow. For

NOMENCLATURE

a	groove height, Fig. 1
A	fluid mixing parameter
d	cylinder diameter
E	enhancement parameter
f	frequency
Gr	Grashof number
h	channel half height, Fig. 1
k	thermal conductivity
l	groove length, Fig. 1
L	groove spacing, Fig. 1
Nu	Nusselt number
Pr	Prandtl number
q	heat flux
Q	volume flow rate
R_c	Reynolds number for self-sustained oscillation
Re	Reynolds number
Re_c	critical Reynolds number
t	time
T	temperature
U	velocity
V	mean velocity.

Greek symbols

Ω_F	dimensionless forcing frequency
Ω_N	dimensionless natural frequency
η	the oscillatory fraction of the flow rate
ν	kinematic viscosity.

$Re < R_c$ a flow disturbance would decay in a damped oscillatory mode with approximately the same frequency as the one dominating the self-sustained oscillations [3].

From the above we conclude that for separated flows there exists a least stable frequency which could be excited not only for Reynolds number above the critical value, but also below it. This suggests that we can exploit this feature by deliberately modulating certain classes of flows at their resonant frequencies, leading to a large-scale lateral mixing and associated augmentation of heat transfer ('resonant enhancement').

Such resonant enhancement is appropriate for systems with naturally occurring, separated flows operated in the subcritical flow range (i.e. without spontaneously occurring self-sustained lateral mixing), e.g. coolant flows in electronic equipment, louvered fins and biomedical devices. Other useful applications might include mixing enhancement of laminar sublayers caused by deliberate modifications of the wall region (i.e. machining microgrooves in the wall) and subsequent modulatory excitation of the corresponding least stable frequency or, in some cases, forcing transition to turbulent flow at lower Reynolds numbers. In the next section we present an example demonstrating main features of the resonant enhancement concept.

2. RESONANT ENHANCEMENT IN A GROOVED CHANNEL

2.1. *Problem description*

We consider two dimensional, fully-developed flow in the periodically grooved channel, shown in Fig. 1. The presence of grooves facilitates separated flow—the principal requirement for deployment of the resonant enhancement concept. The general operating parameters for the system are: the volume flow rate, Q, specified as $Q = \bar{Q}[1+\eta \sin(2\pi ft)]$; the oscillatory fraction of the flow rate; and the oscillatory frequency. The flow conditions are specified by the Reynolds number, $Re = 3/2(Uh/\nu)$; the oscillatory fraction of the flow rate; and the forcing Strouhal number, $\Omega_F = 2/3\ (fh/(U)$.

The thermal conditions for the heat transfer calculations and measurements are uniform heat flux from the grooved wall, an adiabatic flat wall and a fully dveloped temperature profile [4]. We define the groove-averaged Nusselt number, $Nu = qh/\Delta T_b k$, where q is the heat flux from one groove periodicity length divided by its projected area, and ΔT_b is the difference between the wall temperature (averaged along one periodicity length) and the bulk fluid temperature.

Our main objectives are: (a) to demonstrate the existence of a resonant frequency (i.e. for $\eta = 0$ and

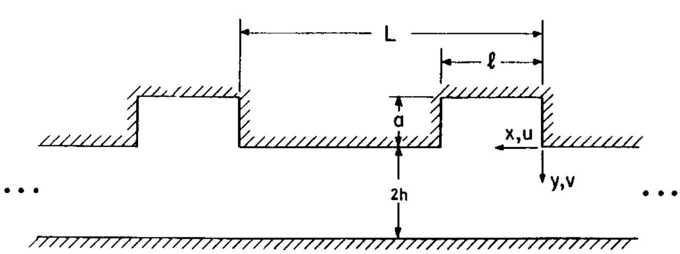


FIG. 1. Periodic, grooved-channel geometry.

$Re > R_c$ the presence of self-sustained oscillation at natural frequency Ω_N); (b) to show that this frequency can be excited for modulated subcritical flows ($Re < R_c$, $\eta > 0$ $\Omega_F = \Omega_N$); and (c) to quantify the resulting enhancement of heat transfer produced by such modulation.

2.2 *Methodology*

In this investigation we use both numerical and experimental techniques. The Navier–Stokes and energy equations are solved numerically in the domain shown in Fig. 1, using the spectral element method [5–8], a high-order solution technique for the Navier–Stokes equations that combines the accuracy of spectral methods with the geometric flexibility of finite-element schemes. The full nonlinear heat transfer calculations have been performed for the case $L = 6.66$, $l = 2.22$, $a = 1.11$ and $h = 1$ (see Fig. 1). Simulations of forced-convection heat transfer are done for Prandtl numbers in the range $1 < Pr < 5$ [4] and the results are extrapolated to $Pr = 7$ [9].

In Fig. 2 we show a diagram of the grooved channel used in the experimental part of this investigation. The flow rate is controlled by a centrifugal mean flow pump and a Scotch yoke oscillatory pump. Water, at 20°C ($Pr = 7$), enters the channel from the right, flows into a development section and enters the grooved test section. For the values of Re investigated in this work, the flow is fully developed by the time it reaches the seventh and eighth grooves, where velocity and heat transfer measurements are made. The ratio of the channel width to the channel half height is 25, and flow visualizations indicate that the structure of the flow is essentially two dimensional. Velocity measurements are made by a hot film probe located at the seventh groove.

Electrical film heaters are located at the grooved wall (including the vertical portions) in the region indicated in Fig. 2. To minimize the buoyancy effects, the ratio Gr/Re^2 (where Gr is the Grashof number) is kept small and the heated surface is placed on the (gravitationally) top surface. Temperature differences between the 16 wall locations shown in Fig. 2 and the fluid bulk temperature are used in calculating the Nusselt number. The dimensionless geometry parameters used for the experimental study are $L = 10.4$, $l = 3.42$, $a = 1.71$ and $h = 1$.

2.3. *Results*

We first show results based on direct numerical simulation. In Fig. 3(a) we plot the perturbation component of streamwise velocity as a function of time for $Re = 1200$ and $\eta = 0$ (no forced modulation). It is evident that for those conditions the flow exhibits a self-sustained flow oscillation. All runs above $Re = 975$ (not shown here) have the same behavior. Figure 3(b) shows the total streamwise velocity after an initial single perturbation for $Re = 800$ and $\eta = 0$. We see that the initial disturbance decays to a steady state in a damped oscillatory mode at frequency

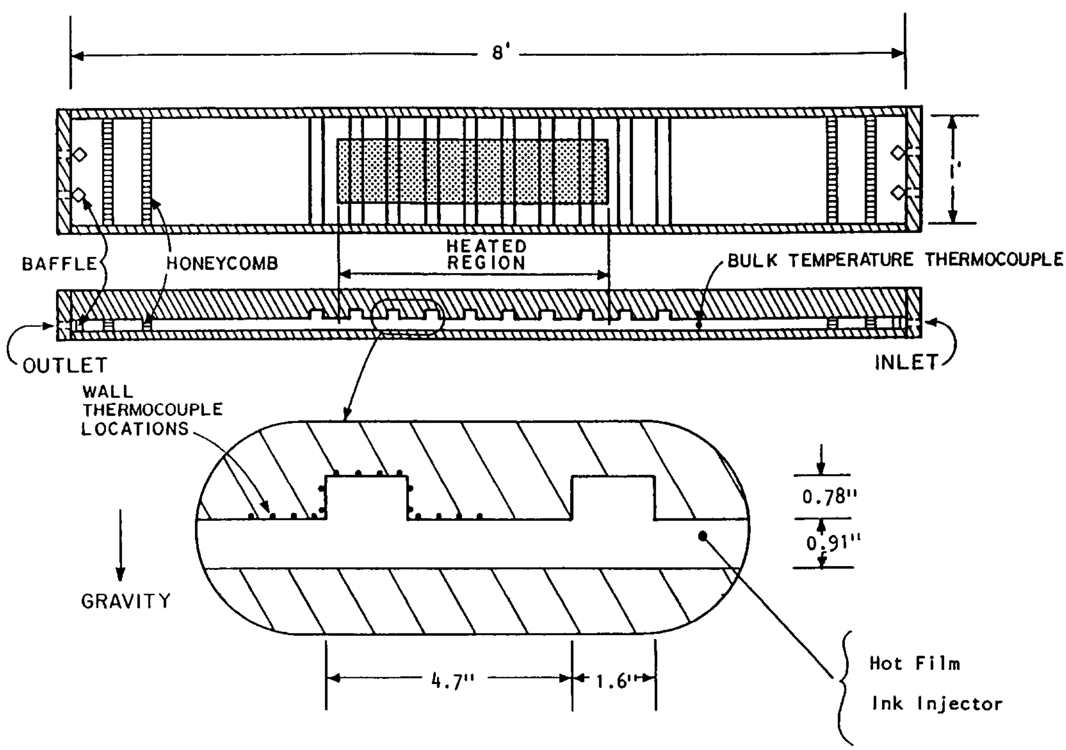
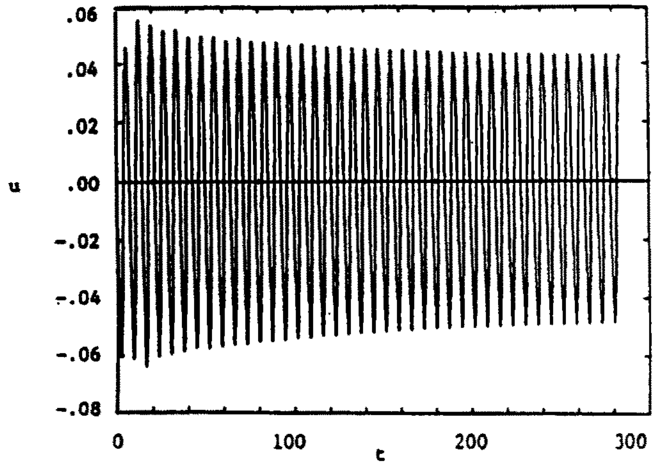
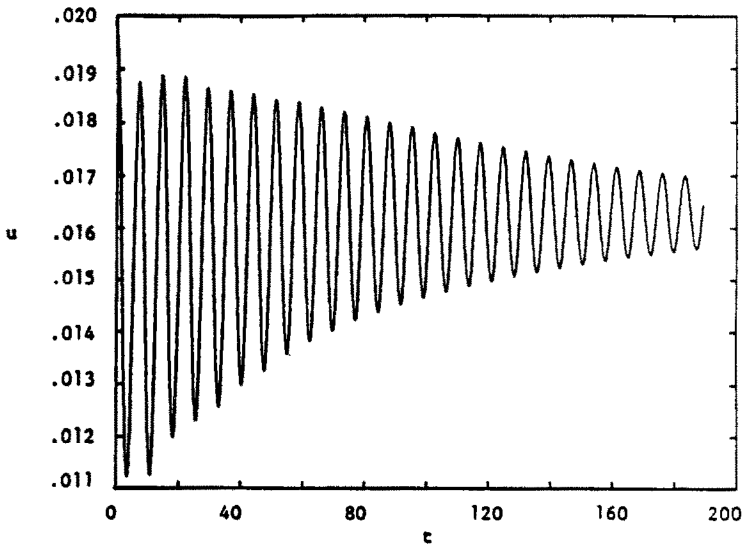


FIG. 2. Test section.



(a)



(b)

FIG. 3. A plot of the velocity u at a typical point (a) $Re = 1200$; flow is not steady, $\Omega_N = 0.141$. (b) $Re = 800$; when perturbed, the flow approaches a steady state in a damped oscillating mode, $\Omega_N \cong 0.141$.

(approximately) matching the self-sustained oscillations [Fig. 3(a)]. Similar behavior persists for other runs below $Re = 975$ (subcritical flows) [3]. The above confirms (for our system) the existence of a natural frequency that is detectable either through self-sustained oscillation for supercritical flows or, as a damped decaying mode, for subcritical flows. The latter also implies that this least stable mode could be excited for $Re < R_c$, suggesting that a modest flow modulation (small y), forced at the natural frequency, could drive strong oscillations for $Re < R_c$. A predictive analytical scheme for determination of the natural frequency for this geometry is presented in ref. [3].

We limit our interest to a subcritical flow for $Re = 525$ and the basic geometry introduced earlier. We start with the base case: steady-state, unmodulated ($\eta = 0$), grooved-channel flow case. In Fig. 4 we plot steady streamlines, isotherms and vertical velocity 'slices' for $Re = 525$ and $Pr = 1$. It is seen that, with the exception of the groove region, the thermal solution is essentially one of conduction or, more precisely, fully-developed internal flow. In the groove region of the domain the temperature distribution is affected by the recirculating flow (and associated boundary layers), however, the effect of the groove vortex does not extend to the channel part of the flow, as evidenced by the insignificant vertical velocities above the cavity.

We proceed by modulating the same system at its natural frequency ($\Omega_N = 0.168$). We seek effects of resonant response of the system by observing the instantaneous velocity and thermal fields during the flow cycle. In Figs. 5–8 we plot the instantaneous

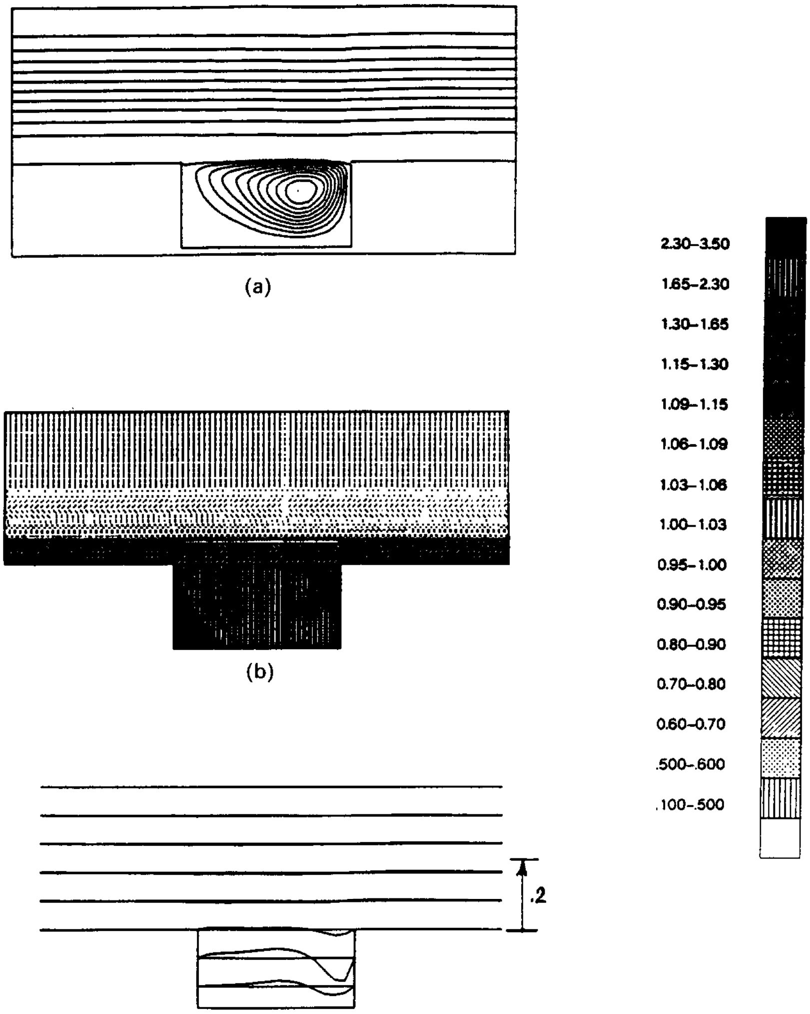


FIG. 4. Plots of the (a) streamlines, (b) isotherms, and (c) vertical velocity slices of the steady flow in the base geometry at $Re = 525$, $Pr = 1$.

streamlines, vertical velocity slices, vorticity contours and isotherms, respectively, during a full cycle for $R = 525$, $\Omega_F = \Omega_N$, $\eta = 0.2$, $Pr = 1$. All the pictures indicate significant mixing between the groove and bulk flow. This is seen in Fig. 5 by the bulging of the groove vortex into the channel; in Fig. 6 by the large vertical velocities at the groove lip; and in Fig. 7 by the motion of packets of 'hot' fluid from the groove, and the motion of packets of 'cold' fluid into the groove.

The effect of forcing frequency on flow excitation and associated transport is measured by two parameters, the first a pointwise amplitude parameter

$$A(\mathbf{x}) = \langle (v(\mathbf{x}, t) - \langle v(\mathbf{x}) \rangle)^2 \rangle^{1/2}$$

corresponding to the magnitude of the fluctuating component of the vertical velocity, the second a transport enhancement parameter

$$E = Nu(Re, \eta, \Omega_F, Pr) / Nu(Re, \eta = 0, \ldots, Pr).$$

In Fig. 8 we plot A at the point in the groove shear layer as a function of Ω_F at $R = 525$, $\eta = 0.2$. It is seen that the magnitude of the fluctuating component of the vertical velocity has its maximum at $\Omega_F = \Omega_N$, indicating the strong resonant nature of the response of the system. In Fig. 9 we plot the enhancement parameter E obtained numerically for $Re = 525$, $Pr = 7$, $\eta = 0.2$ and the base geometry and, in Fig. 10, the same parameter obtained experimentally for the similar (but not identical) experimental geometry and the same flow condition. We see that the enhancement ratio, too, exhibits strong resonant characteristics, and that its maximum value, in the case of the experimental geometry, approaches a respectable factor of 3.

We conclude that the large-scale mixing seen in Figs. 6–9 clearly confirms the presence of resonant response during excitation of flows 'close' to oscillatory instability and that this, together with the

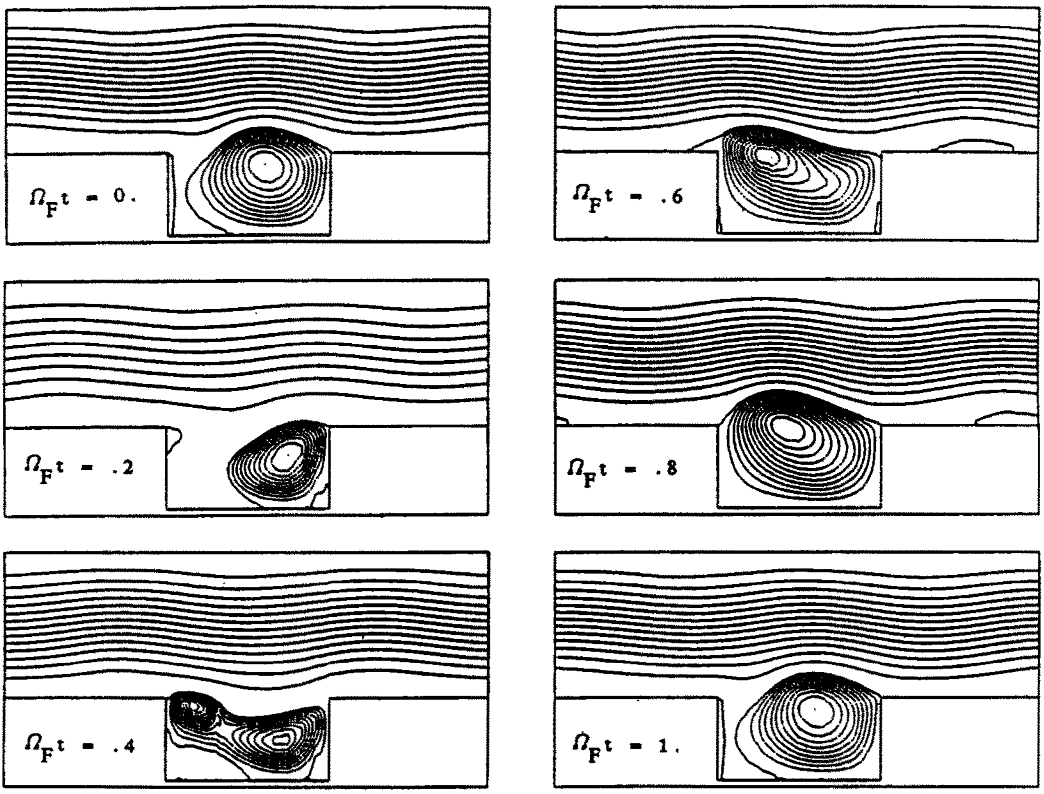


FIG. 5. A plot of the instantaneous streamlines during the flux cycle at $Re = 525$, $\eta = 0.2$, $\Omega_F = 0.168$ $(= \Omega_N)$.

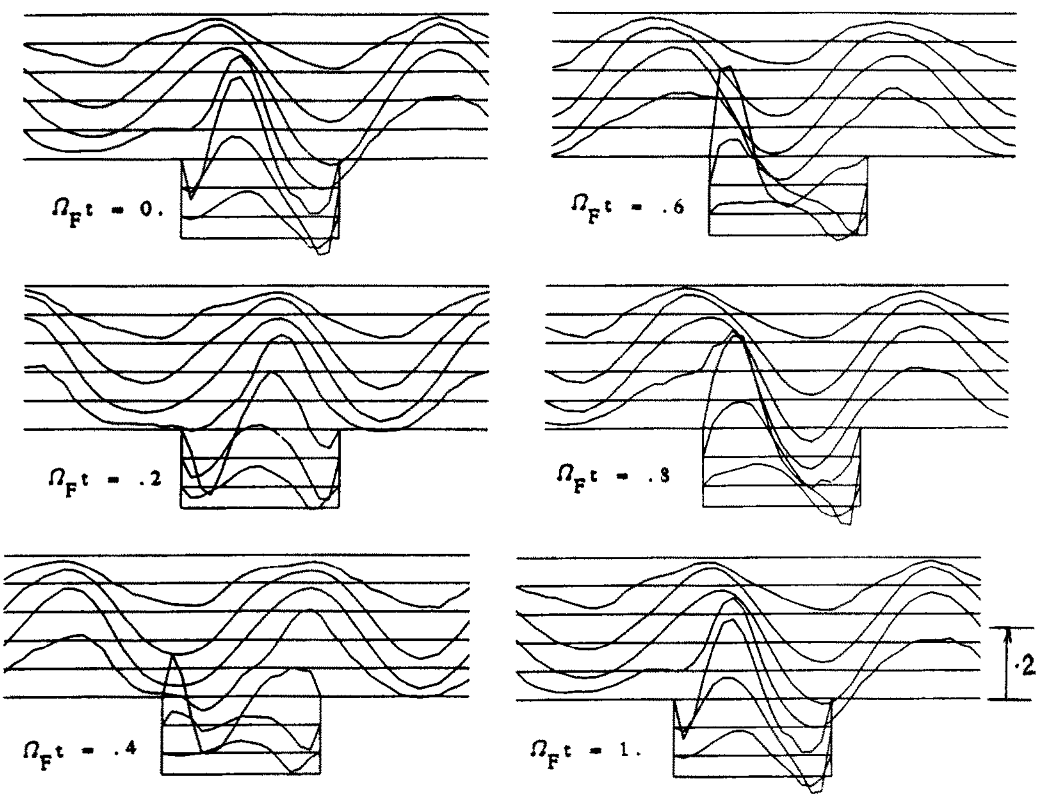


FIG. 6. A plot of the instantaneous vertical velocity slices during the flow cycle at $Re = 525$, $\eta = 0.2$. $\Omega_F = \Omega_N = 1.68$.

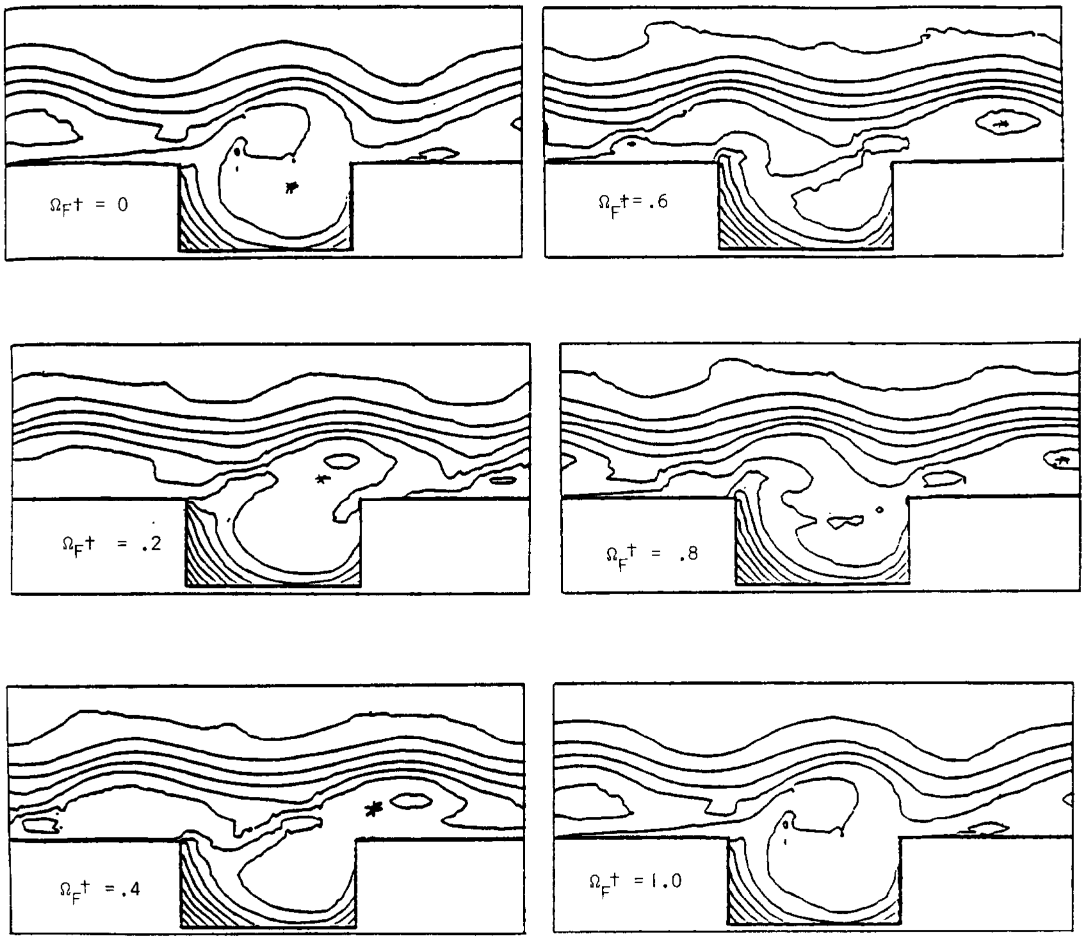


FIG. 7. A plot of the isotherms during one cycle at $Re = 525$, $\eta = 0.2$, $\Omega_F = \Omega_N$, $Pr = 1$.

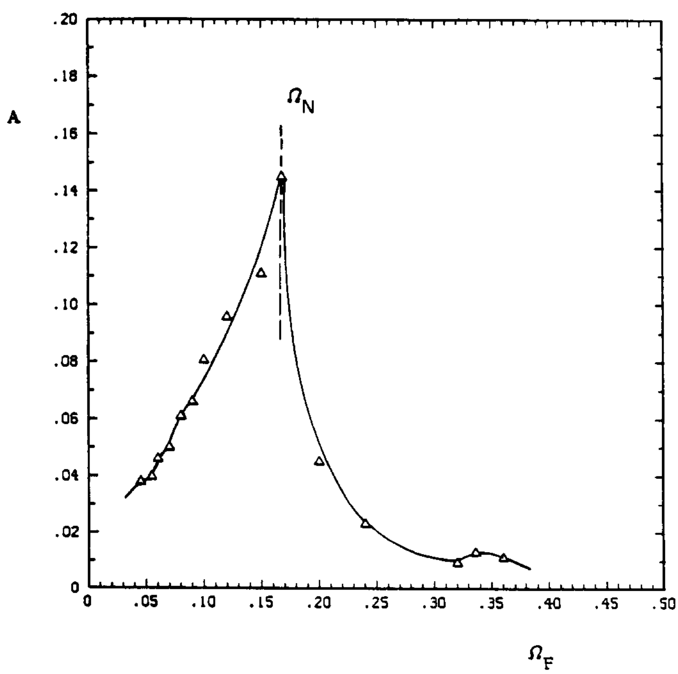


FIG. 8. A plot of the amplitude parameter A, at a point in the groove shear layer as a function of forcing frequency at $\eta = 0.2$, $Re = 525$.

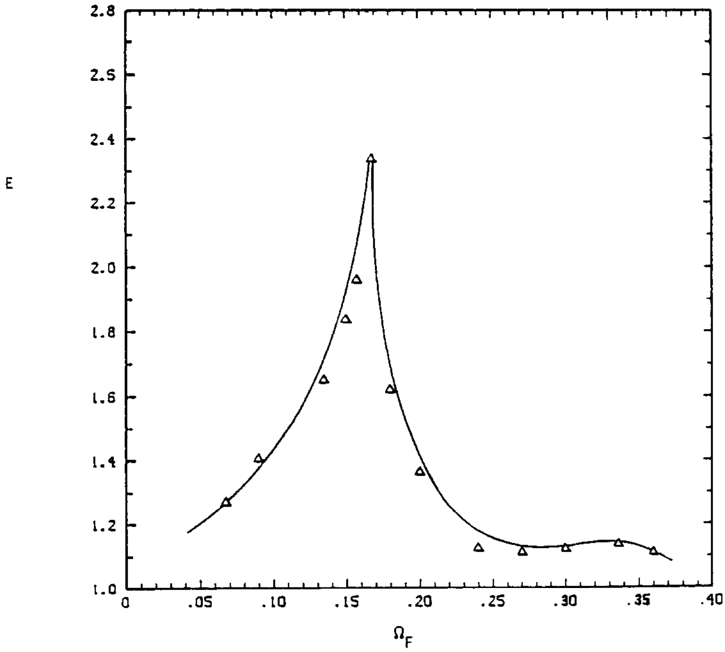


FIG. 9. A plot of the numerically calculated heat transfer enhancement, E, as a function of forcing frequency, for base geometry at $Re = 525$, $\eta = 0.2$, $Pr = 7$ (extrapolated).

resulting dramatic heat transfer enhancements shown in Figs. 9 and 10, suggest that this powerful augmentation mechanism has potential for further significant utilizations.

The important question of pressure drop for this geometry was considered in ref. [10]. The initial results based on numerical simulation indicates that at the onset of large-scale mixing (produced either by a flow modulation or spontaneously for $Re > R_c$), the pressure drop increases dramatically but the relative increase was still less than the corresponding increase in heat transfer.

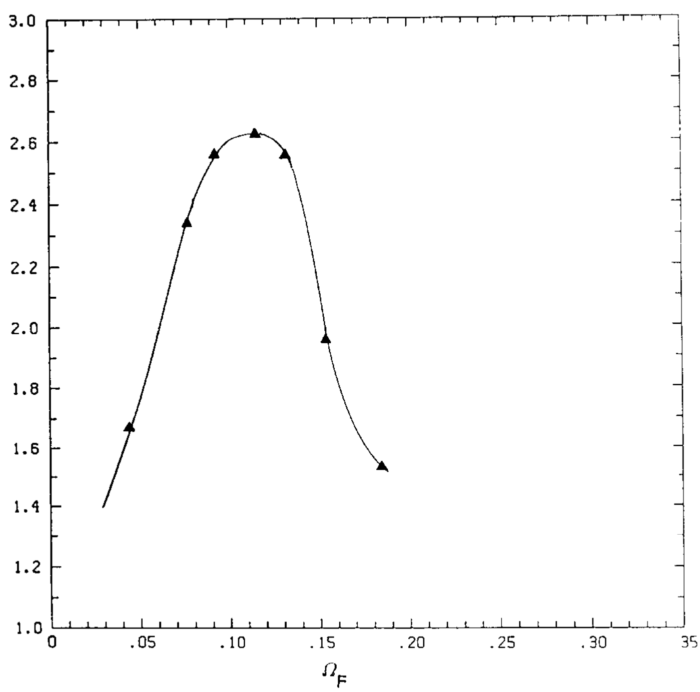


FIG. 10. A plot of the experimentally measured heat transfer enhancement, E, as a function of forcing frequency for experimental geometry at $Re = 525$ and $\eta = 0.2$.

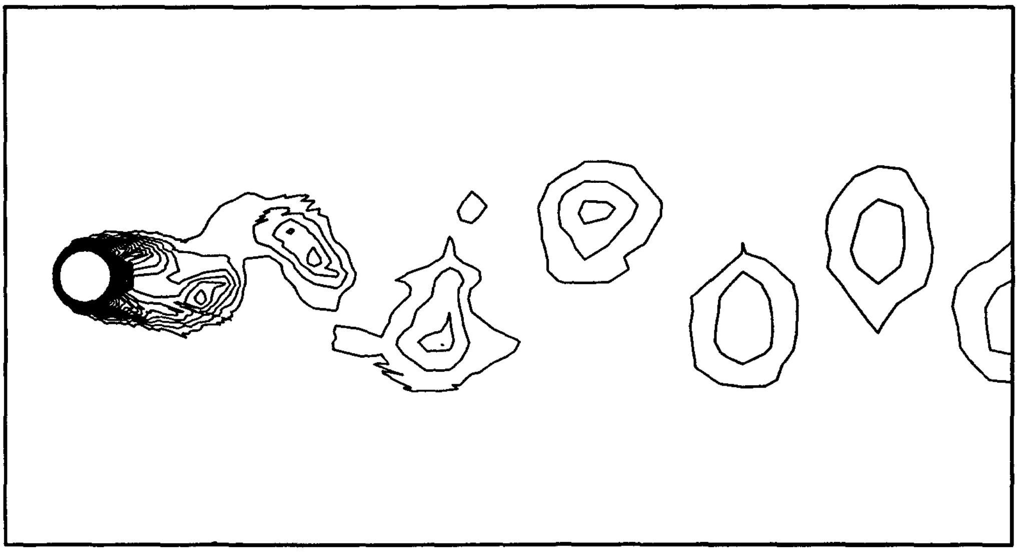
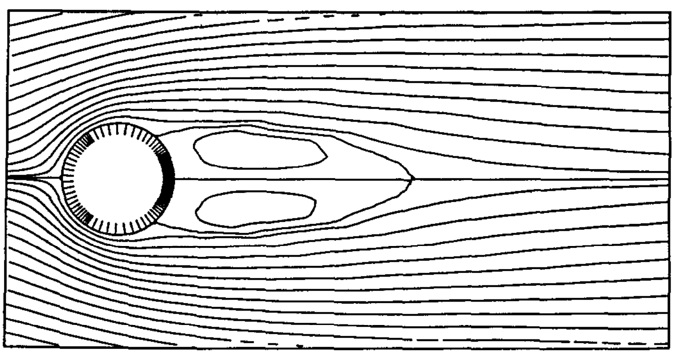


FIG. 11. A plot of instantaneous isotherms for flow across cylinder at $Re = 50$, $\eta = 0.0$, $Pr = 2$.

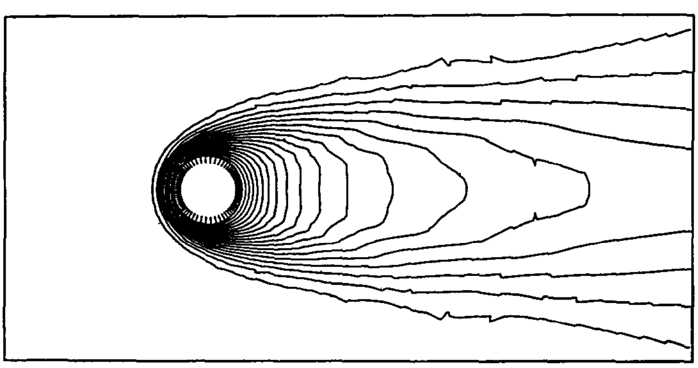
3. RESONANT RESPONSE IN EXTERNAL CROSS FLOW AROUND CYLINDER

In this section we consider briefly an external cross flow over a long circular cylinder in order to show that the resonant nature of the separated flow behavior is not unique to the grooved channel geometry. The results in this section are obtained by numerical simulation with the methodology already described [5–8].

In Fig. 11 we plot the instantaneous isotherms for unmodulated flow, $\eta = 0$, at $Re = v_{\infty}d/\nu = 50$. We find for this case, not surprisingly, an unsteady behavior with periodic vortex shedding at the normalized natural frequency (Strouhal number) of 0.146



(a)



(b)

FIG. 12. A plot of (a) streamlines and (b) corresponding isotherms for flow across cylinder at $Re = 30$ and $\eta = 0.0$.

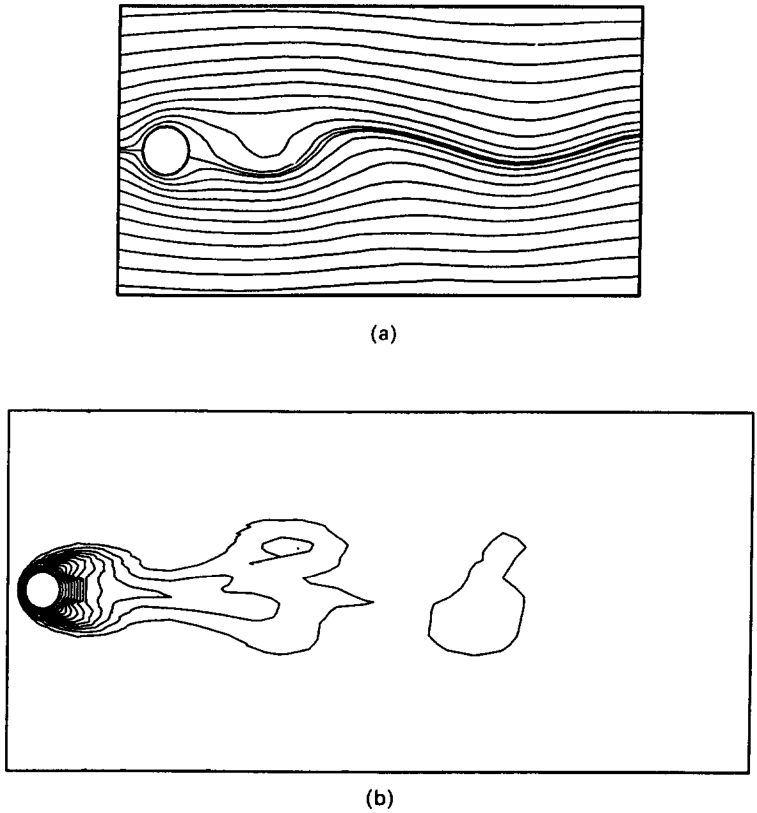


FIG. 13. A plot of instantaneous (a) streamlines and (b) isotherms for flow across cylinder at $Re = 30$, $\eta = 0.3$ and $Pr = 2$.

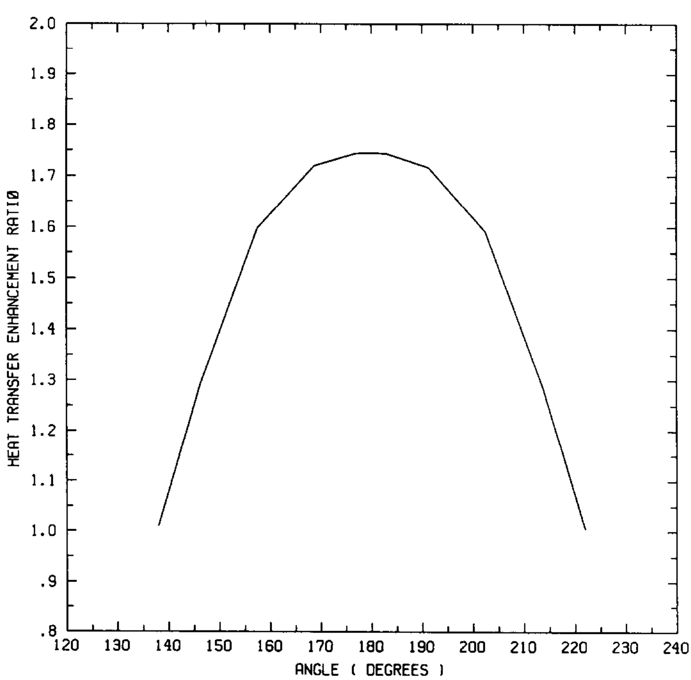


FIG. 14. A plot of heat transfer enhancement in separated region for flow across cylinder at $Re = 30$, $\eta = 0.3$ and $Pr = 2$.

in agreement with the experimental evidence [11]. We interpret the observed behavior as a manifestation of the self-sustained oscillations driven by the instabilities of the shear layer formed in the separated region. We further conclude that the flow is obviously supercritical at this Reynolds number.

We then proceed with a numerical simulation of a subcritical flow, choosing $Re = 30$, $\eta = 0$. In Figs. 12(a) and (b) we plot for this case the streamlines and isotherms, respectively. As expected, the stable flow is steady, as $Re = 30 < R_c \approx 40$. Next, the subcritical flow considered above is modulated ($\eta = 0.3$) at the natural frequency found in the first example ($Re = 50$), and the resulting instantaneous streamlines and isotherms are plotted in Figs. 13(a) and (b). We see clearly a resonant response, corresponding to periodic vortex shedding in a 'naturally' subcritical flow. We close by plotting the ratio of respective Nusselt numbers (local, time average) for the last two cases ($\eta = 0$ case in denominator) along the heated surface (constant wall temperature, $Pr = 2$) downstream of the separation point (135° from the front stagnation point) Fig. 14. A significant (resonant) heat transfer augmentation is evident.

4. CONCLUSIONS

For the two separated flows considered—fully developed flow in a periodically grooved channel, and external cross flow over a cylinder—we observe a similar resonant behavior: self-sustained oscillations for $Re > R_c$, the existence of natural frequency, and an ability to excite this frequency for $Re < R_c$, leading to a large-scale mixing and significant heat transfer enhancement. We conclude that the resonant behavior in separated flows is a common phenomenon which offers broad, and as yet unexplored, opportunities for controlled application in a wide range of transport processes.

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EXPLOITATION DES INSTABILITES HYDRODYNAMIQUES; ACCROISSEMENT DU TRANSFERT THERMIQUE PAR RESONNANCE

Résumé—On introduit le concept de l'accroissement du transfert de chaleur basé sur l'excitation des instabilités de la couche de cisaillement présentes dans les écoulements internes séparés. L'exploitation des instabilités naturelles demande: la création d'un système avec séparation d'écoulement, la détermination d'une fréquence de résonnance et l'excitation de cette fréquence avec une modulation appropriée. Les mouvements résultants à grande échelle conduisent à un mélange latéral et à un accroissement très grand du transfert thermique. La méthode est applicable en écoulements laminaires ou turbulents. On présente des résultats expérimentaux et numériques pour un écoulement subcritique en canal rainuré et pour un écoulement frontal autour d'un cylindre. Dans le cas du canal rainuré, on observe un accroissement du transfert thermique par triplement lorsque l'écoulement est modulé à la fréquence naturelle du système.

AUSNUTZUNG HYDRODYNAMISCHER INSTABILITÄTEN ZUR ERHÖHUNG DER WÄRMEÜBERTRAGUNG

Zusammenfassung—Wir stellen hier das Konzept der Erhöhung der Wärmeübertragung durch Resonanz vor, das sich auf Anregung der Scherungsschicht-Instabilitäten stützt, die in innerlich getrennten Strömungen vorhanden sind. Die Ausnutzung von natürlichen Instabilitäten erfordert: Herstellung eines Systems mit getrennter Strömung, Bestimmung der Resonanzfrequenz des Systems und die Anregung dieser Frequenz durch eine entsprechende Einstellung. Die sich ergebenden Bewegungen führen zu einer spürbaren Quervermischung und einer entsprechenden Verbesserung der Wärmeübertragung. Diese Methode ist sowohl bei laminarer als auch turbulenter Strömung anwendbar. Experimentelle und numerische Ergebnisse für eine unterkritische Kanalströmung und einen querangeströmten Zylinder werden vorgestellt. Im Falle der Kanalströmung wird ein Anstieg der Wärmeübertragung bis zum dreifachen Wert beobachtet, wenn die Strömung auf die natürliche Systemfrequenz eingestellt wird.

ИСПОЛЬЗОВАНИЕ ГИДРОДИНАМИЧЕСКИХ НЕУСТОЙЧИВОСТЕЙ ПРИ РЕЗОНАНСНОМ УСИЛЕНИИ ТЕПЛООБМЕНА

Аннотация—В статье вводится понятие резонансного усиления теплообмена за счет возбуждения в пограничном слое неустойчивостей, присущих внутренним отрывным течениям. Для использования известных неустойчивостей требуется создать систему с отрывным потоком, определить ее резонансную частоту и возбудить эту частоту посредством соответствующей модуляции. Возникающие крупномасштабные течения приводят к существенному поперечному перемешиванию и, соответственно, резкому усилению теплообмена. Предлагаемый метод можно использовать как при ламинарных, так и при турбулентных течениях. Приводятся экспериментальные и численные результаты для докритического потока в рифленом канале и поперечного обтекания цилиндра. Для рифленого канала при модуляции потока с собственной частотой системы наблюдается трехкратное усиление теплообмена.